

# A Note on Hamilton Cycles

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## Abstract

If  $G$  is a more than one tough graph on  $n$  vertices with  $\delta \geq \frac{n}{2} - a$  for a given  $a > 0$  and  $n$  is large enough then  $G$  is hamiltonian.

Key words: Hamilton cycle, minimum degree, toughness.

## 1 Introduction

Only finite undirected graphs without loops or multiple edges are considered. We use  $n$ ,  $\delta$ ,  $c$  and  $\tau$  to denote the number of vertices (order), the minimum degree, circumference and the toughness of a graph, respectively. Let  $s(G)$  denote the number of components of a graph  $G$ . A graph  $G$  is  $t$ -tough if  $|S| \geq ts(G \setminus S)$  for every subset  $S$  of the vertex set  $V(G)$  with  $s(G \setminus S) > 1$ . The toughness of  $G$ , denoted  $\tau(G)$ , is the maximum value of  $t$  for which  $G$  is  $t$ -tough (taking  $\tau(K_n) = \infty$  for all  $n \geq 1$ ). A good reference for any undefined terms is [2].

The earliest degree condition for a graph to be hamiltonian is due to Dirac [3].

**Theorem A [3].** Every graph with  $\delta \geq \frac{1}{2}n$  is hamiltonian.

Jung [4] proved that the minimum degree bound  $\frac{1}{2}n$  in Theorem A can be slightly relaxed when  $n \geq 11$  and  $\tau \geq 1$ .

**Theorem B [4].** Every graph with  $n \geq 11$ ,  $\tau \geq 1$  and  $\delta \geq \frac{n}{2} - 2$  is hamiltonian.

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This bound  $\frac{n}{2} - 2$  itself was lowered to  $\frac{n}{2} - 3.5$  under stronger conditions  $n \geq 30$  and  $\tau > 1$ .

**Theorem C [1].** Every graph with  $n \geq 30$ ,  $\tau > 1$  and  $\delta \geq \frac{n}{2} - 3.5$  is hamiltonian.

In this note we show the following.

**Theorem 1.** Let  $G$  be a graph with  $\tau > 1$  and  $\delta \geq \frac{n}{2} - a$  for a given  $a > 0$ . Then  $G$  is hamiltonian if  $n$  is large enough.

The proof of Theorem 1 easily follows from the following theorem due to Jung and Wittmann [5].

**Theorem D [5].** In every 2-connected graph,  $c \geq \min\{n, (\tau + 1)\delta + \tau\}$ .

**Proof of Theorem 1.** Let  $\tau = 1 + \epsilon$  for some  $\epsilon > 0$ . If

$$n \geq \frac{4a - 2}{\epsilon} + 2a - 2 \tag{1}$$

then equivalently

$$(2 + \epsilon) \left( \frac{n}{2} - a + 1 \right) - 1 \geq n$$

and by Theorem D,

$$c \geq (\tau + 1)\delta + \tau \geq (2 + \epsilon) \left( \frac{n}{2} - a + 1 \right) - 1 \geq n.$$

So, if (1) then  $G$  is hamiltonian. Theorem 1 is proved.

## References

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